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MODELING OF THE RETURN OF TURBULENCE TO AN ISOTROPIC STATE

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The study models a correlation that involves pressure oscillations and that determines the return of turbulence to an isotropic state. The modeling is carried out via a direct comparison of approximations put forward previously with correlations resulting from a numerical calculation of the time-dependent Navier–Stokes equations. The form of the approximations and the empirical coefficients are determined. The results can be employed for constructing new turbulence models of the second order.

Introduction. The solution of applied problems of hydrodynamics and heat and mass transfer in turbulent fluid flows requires that the components of the Reynolds stress tensor and the vector of the heat flux be ascertained. Generally, one or another turbulence model is used to this end. The first-generation turbulence models relied on the concept of the turbulent viscosity, which was obtained from empirical equations. Later, wide popularity was gained by the $K-\varepsilon$ model, which also relied on the concept of the turbulent viscosity, whose functional dependence on the turbulent kinetic energy and the dissipation rate follows from a relation that is set up so that its dimension matches that of the viscosity. Detailed investigations of the first-generation turbulence models revealed an essential shortcoming in them, viz., nonuniversality. In other words, appropriate model coefficients must be selected for predicting each new type of flow in order that the predicted results are in favorable agreement with pertinent experimental data. The second-generation models aim at developing a universal turbulence model. A way to construct it was proposed in [1] in the early 70s. The essence of this approach lies in that the Reynolds stresses and the heat fluxes should be obtained from exact equations for second-order single-point moments:

$$\frac{DR_{ij}}{Dt} = F_{ij} + P_{ij} + \Phi_{ij} - 2\varepsilon_{ij} + D_{ij}\,, \qquad (1)$$

$$\frac{D\Gamma_i}{Dt} = F_{(\tau)i} + P_{(\tau)i} + \Phi_{(\tau)i} - \varepsilon_{(\tau)i} + D_{(\tau)i}\,. \qquad (2)$$

Accoroding to the authors' plan, use of Eqs. (1) and (2) will permit a conversion from intuitive modeling based on some physical analogies to modeling based on exact equations for second-order single-point moments. Furthermore, employment of exact equations and physically justified approximations for unknown correlations is a reliable basis for obtaining universal equations that can be used with confidence for describing turbulent transfer.

Equations (1) and (2) accurately describe convective transfer and generation processes. The term containing oscillations of the external force depends on its type. For stratified flows, it is determined exactly. Dissipation and diffusion terms and correlations containing pressure oscillations should be modeled, since they involve unknown correlations of higher order. The state of studies in this direction is as follows.

Study [1] generalized the results of investigations conducted before 1974 and advanced a complete turbulence model of the second order that, with some modifications, was used widely in numerical calculations of turbulent flows in the next 20 years. The correlation Φ_{ij} was modeled using the earlier Rott model and a model devised by then for describing the interaction of the mean shear with velocity oscillations. Dissipation processes were determined on the basis of the Kolmogorov hypothesis of local isotropy of the velocity field. Diffusion terms

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were modeled via relations based on the hypothesis that the turbulent diffusion is of a gradient nature. Subsequent studies [2-5] modeled the correlation Φ_{ij} using a method of invariant modeling and realizability conditions for a turbulence model of the second order. Study [6] compared experimental data with results predicted using models for the correlation Φ_{ij} and D_{ij} worked out by then. Study [7] proposed a model for the correlation $\Phi_{(\tau)i}$, and study [8] analyzed the efficiency of various turbulence models of the second order and described the merits and demerits of individual models and of turbulence models of the second order as a whole.

Turbulence models of the second order were tested in numerical calculations of jet flows, boundary-layer flows, various-geometry duct flows, and stratified and swirling flows. The calculated results demonstrated that such models are fairly universal and allow the description of a variety of effects falling outside the scope of the $K-\varepsilon$ model and other models based on the concept of the turbulent viscosity. It can be noted here that the turbulence models of the second order can describe the effect of buoyant forces on the flow structure, the influence of the streamline curvature on the boundary-layer flow, and the appearance of secondary flows in rectangular ducts. Simultaneously, drawbacks of the turbulence models of the second order were found. An analysis of the latter showed that advance in this area is linked to the development of novel approaches to the methodology of identifying the form of approximate expressions for unknown correlations and the ways of determining the system of empirical coefficients entering the approximate expressions. It follows from published works that the most difficult to model is the correlation of strain rates with pressure oscillations Φ_{ij} that characterizes the redistribution of oscillatory-motion energy among the components of the Reynolds stress tensor.

The current study sought to determine the coefficients and the form of approximations for the components of the correlation Φ_{ij} that describes the return of turbulence to an isotropic state. Particular emphasis was placed on the special features of this process in the strongly anisotropic turbulence frequently encountered in stratified, rotational, and magnetohydrodynamic flows. As distinct from studies published previously, the form of the approximation and the empirical coefficients were determined via a direct comparison of the general form of the approximation for the sought correlation with numerical results for the time-dependent Navier–Stokes equations. The primary focus was on searching for methods of correctly allowing for all chief factors affecting the return of turbulence to an isotropic state.

1. Methods of Modeling the Correlation Φ_{ij}. When modeled, this correlation is generally represented as a sum of three terms:

$$\Phi_{ij} = \Phi_{(1)ij} + \Phi_{(2)ij} + \Phi_{(3)ij},$$

where $\Phi_{(1)ij}$ depends only on the interaction of the velocity oscillations and reflect the approach of the field of velocity oscillations to an isotropic state, $\Phi_{(2)ij}$ relates to the interaction of the mean velocity shear with the velocity oscillations, and $\Phi_{(3)ij}$ accounts for the wall effect.

The term $\Phi_{(1)ij}$ is a symmetric nondivergent tensor. In isotropic turbulence, it is equal to zero. In anisotropic turbulence, $\Phi_{(1)ij}$ defines the energy transfer among oscillation components. The degree of turbulence anisotropy can be characterized with the aid of the tensor of Reynolds stress anisotropy $b_{ij} = (R_{ij}/2K) - 1/3\delta_{ij}$ and the tensor of the anisotropy of the dissipation rate for Reynolds stresses $d_{ij} = (\varepsilon_{ij}/\varepsilon) - 1/3\delta_{ij}$. The quantities written also are symmetric tensors of the second rank and are equal to zero in isotropic turbulence. For anisotropic turbulence developing in a flow with no velocity shear, the turbulent-energy generation is equal to zero, and the term of the energy redistribution among oscillation components is $\Phi_{ij} = \Phi_{(1)ij}$. In this case, the rate of energy transfer among the components of the Reynolds stresses should increase with b_{ij}. Therefore it is reasonable to suppose that the correlation defining the approach of turbulence to an isotropic state can be represented as

$$\frac{\Phi_{(1)ij}}{\varepsilon} = F(b_{ij}).$$

Study [5] showed that the expression for $\Phi_{(1)ij}$ reduces to a relatively simple form quadratic in b_{ij}:

$$\frac{\Phi_{(1)ij}}{\varepsilon} = -C_1 b_{ij} + C_2\left(b_{ij}^2 + \frac{2}{3} II\delta_{ij}\right), \tag{3}$$

where, in conformity with the Cayley–Hamilton theorem, the equation coefficients can be expressed as a power series:

$$C_\alpha = C_{\alpha 0} + C_{\alpha 1}\, II + C_{\alpha 2}\, III\,. \tag{4}$$

Attention is turned next to a special feature of the approximation put forward in [5]. The general form of approximation (3) is simple; however, the coefficients entering expression (3) are specified via a series. Thus, all the complications of modeling the return of turbulence to an isotropic state are associated precisely with determination of the coefficients of series (4).

Published studies generally employ simpler approximations that can be regarded as special cases of relation (3):

model [1]: $C_1 = 3.0$, $C_2 = 0$;
model [2]: $C_1 = \beta$, $C_2 = 0$;
model [3]: $C_1 = (3.4 + 1.8P/\varepsilon)$, $C_2 = 4.2$;

$$\beta = 2 + \frac{F}{9}\exp\left(-\frac{7.77}{\sqrt{\mathrm{Re}_t}}\right)\left[\frac{72}{\sqrt{\mathrm{Re}_t}} + 80.1\ln\left(1 + 62.4\,(2II + 6.9III)\right)\right],$$

$$F = 1 + 9II + 27III\,,\quad \mathrm{Re}_t = 4\,\frac{K^2}{9\varepsilon\nu}\,.$$

As will be demonstrated below, the coefficients of the approximation for $\Phi_{(1)ij}$ reported in [1-3] inadequately describe the distributions of this correlation obtained in [9] using the method of direct numerical simulation of the time-dependent Navier–Stokes equations. Therefore, practical use of the model represented by Eqs. (3) and (4) calls for a more accurate determinination of the coefficients and the form of their dependence on the invariants II and III.

2. Initial Data for Approximating $\Phi_{(1)ij}$. Basically, there are three ways of obtaining the empirical coefficients, viz.:

1) the use of experimental data;

2) determination of the empirical coefficients and the form of the approximations for the unknown correlations by comparing measurement data with results of numerical calculations carried out using various approximation formulas for different values of the coefficients;

3) application of results of a direct numerical simulation for the time-dependent Navier–Stokes equations.

Experimental data are reviewed in [10, 11]. To date, all components of the Reynolds stress, the dissipation rate, the convection terms, the generation of turbulent kinetic energy, and the diffusion terms have been measured. The components of the tensor dissipative function and the terms containing pressure oscillations were not determined for lack of appropriate measuring procedures. As a consequence, only the difference of terms $(\Phi_{ij} - 2\varepsilon_{ij})$ can be obtained from results for uniform turbulence. The quantity Φ_{ij} was determined on the assumption that the dissipation process is isotropic. The coefficients thus established were tested by comparing numerical results with experimental data for various turbulent flows. Results of calculations showed [8] that the approximations obtained for the dissipation terms and the terms involving the correlations of pressure oscillations cannot be recognized as satisfactory.

Numerous attemps to indirectly model unknown correlations by selecting empirical coefficients and to subsequently compare experimental data for averaged quantities with numerical results were not successful, which was corroborated by materials submitted to the Stanford Conference of 1981. In this connection, it is reasonable to follow the third path, viz., to utilize results of a direct numerical simulation for approximating unknown correlations.

Studies [9, 11, 12] reported results of a direct numerical simulation (DNS) of the time-dependent Navier–Stokes equations for a number of simple two-dimensional flows at moderate turbulent Reynolds numbers (Re_t = 180 and 395). Fairly dense grids ($2\cdot 10^6$ and $4\cdot 10^6$ points) were used in the calculations, which allowed

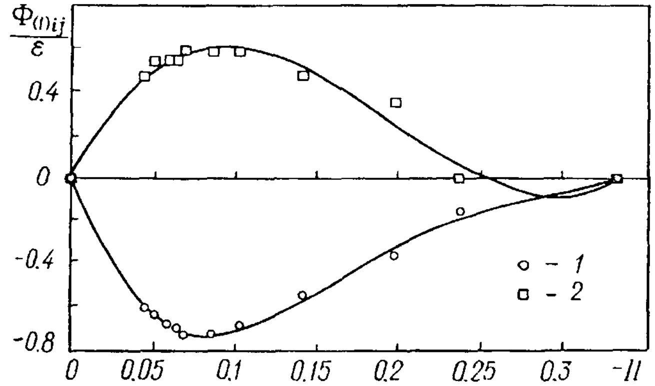

Fig. 1. DNS data for components of the tensor $\Phi_{(1)ij}/\varepsilon$: 1) $\Phi_{(1)xx}/\varepsilon$, 2) $\Phi_{(1)yy}/\varepsilon$.

resolution of all scales essential to describing turbulence. The DNS results afford information on the structure of turbulence that cannot be established experimentally. From this standpoint, DNS results are a valuable tool for constructing semiempirical turbulence models.

3. Results of Modeling the Process of Return to Isotropy. The coefficients C_i in approximations (3) and (4) are determined using DNS results [9]. Equation (3) in a Cartesian system takes the form

$$\frac{\Phi_{(1)xx}}{\varepsilon} = - C_1 b_{xx} + C_2 \left(b_{xx}^2 + b_{xy}^2 + \frac{2}{3} II \right) , \quad (5)$$

$$\frac{\Phi_{(1)yy}}{\varepsilon} = - C_1 b_{yy} + C_2 \left(b_{yy}^2 + b_{xy}^2 + \frac{2}{3} II \right) . \quad (6)$$

Figure 1 shows the distributions of $\Phi_{(1)xx}/\varepsilon$ and $\Phi_{(1)yy}/\varepsilon$ obtained in [9] for a steady-state flow in a plane duct. It should be noted that, in isotropic turbulence, invariant II is equal to zero, and the energy transfer among oscillation components ceases, i.e., $\Phi_{(1)xx}(\varepsilon)$ and $\Phi_{(1)yy}/\varepsilon$ are also equal to zero. Less evident is the behavior of these functions in the case of strongly anisotropic turbulence, where the absolute value of invariant II is larger than the values observed in the wall region of the boundary layer. To elucidate this question, we examine a diagram of turbulence states (see Fig. 2). It was analyzed for the first time in [13], where it was shown that all turbulence states should be within the outline depicted in the figure. Point 1 corresponds to isotropic turbulence. The basic parameters in an isotropic state have the following values: $b_{ij} = 0$, $II = 0$, $III = 0$, and $F = 1$. Point 2 corresponds to one-dimensional turbulence for which $II = -1/3$, $III = 2/27$, and $F = 0$. Curves 1-2 and 1-3 are defined by the equation $III^2 + 4\,II^3/27 = 0$ and correspond to axisymmetric turbulence. Straight line 2-3 is described by the equation $F = 1 + 9\,II + 27\,III = 0$ and contains a combination of two-dimensional states of turbulence. The points in Fig. 2 reflect states of turbulence for a developed flow in a plane-parallel duct at various distances from the wall. Near the wall, the normal component of velocity oscillations is small, and therefore the points lie near a straight line describing two-dimensional turbulence. With increasing distance from the wall, the state of turbulence approaches point 2. At $Y^+ = 3.5$, the state of turbulence is closest to one-dimensional. At larger distances from the wall, the turbulence approaches an isotropic state. It should be noted that the one-dimensional state of turbulence is a hypothetical state with only one component of velocity oscillations. As a result, the energy transfer among the components of the Reynolds stress tensor ceases, i.e., $\Phi_{ij} \to 0$. On the other hand, the dissipation processes proceed when a one-dimensional state is approached, i.e., the dissipation rate is other than zero in this case. Hence, $\Phi_{(1)ij}/\varepsilon \to 0$ and $\Phi_{(2)ij}/\varepsilon \to 0$ as $II \to -1/3$. Thus, the distributions of $\Phi_{(1)xx}/\varepsilon$ and $\Phi_{(1)yy}/\varepsilon$ plotted in Fig. 1 should tend to zero when invariant II approaches its limiting value, equal to $-1/3$, and invariant $III \to 2/27$. Using this condition, from Eqs. (5) and (6) we derive an additional relation for the sought coefficients:

$$C_{10} = \frac{1}{3} C_{11} + \frac{2}{27} C_{12} = \frac{1}{3} \left(C_{20} - \frac{1}{3} C_{21} + \frac{2}{27} C_{22} \right) . \quad (7)$$

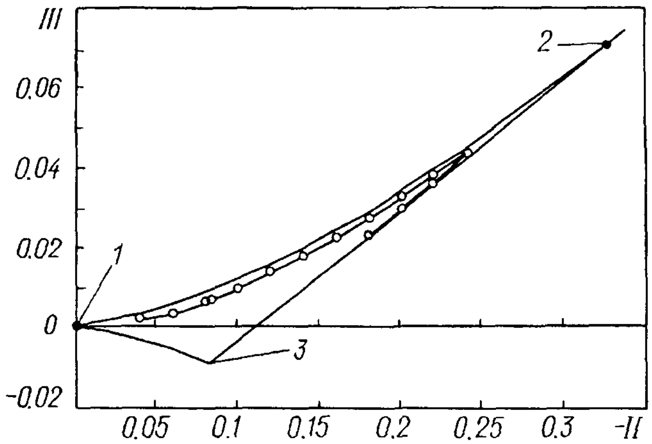

Fig. 2. Diagram of states of turbulence: 1) the point of an isotropic state of turbulence; 2) one-dimensional turbulence; straight line 2-3 corresponds to two-dimensional turbulence; curves 1-2 and 1-3, to axisymmetric turbulence; dots denote DNS data.

It should be pointed out that a similar result was obtained in [5] by analyzing the realizability conditions of a turbulence model of the second order. A model is regarded as realizable if it guarantees that the quantities that should be nonnegative, for example, $R_{\alpha\alpha}$, will always remain nonnegative, and the absolute values of the correlation coefficients will not be larger than unity. Study [5] considered a simpler case of Eq. (4) where the coefficients C_1 and C_2 were assumed to be constants and, from the realizability conditions, the relation $C_1 = C_2/3$ was obtained, which for these conditions follows from expression (7).

The coefficients $C_{\alpha\beta}$ were identified using Eqs. (4)-(7) and DNS data (Fig. 1). Processing of the DNS data yielded the following values of the sought coefficients:

$$C_{10} = 5.48\,;\quad C_{11} = 38.5\,;\quad C_{12} = 115.6\,;\quad C_{13} = 0\,;$$

$$C_{20} = 3.6\,;\quad C_{21} = C_{22} = C_{23} = 0\,.$$

The mean error in describing the distributions of $\Phi_{(1)xx}/\varepsilon$ and $\Phi_{(1)yy}/\varepsilon$ using the written coefficients is 2.2%. The results of modeling the return of turbulence to an isotropic state can be written in a simpler form:

$$\frac{\Phi_{(1)ij}}{\varepsilon} = -C_1 b_{ij} + C_2\left(b_{ij}^2 + \frac{2}{3} II\,\delta_{ij}\right), \tag{8}$$

$$C_1 = C_{10f} + C_{11f}F\,,\quad C_2 = 2C_{10f}\,,\quad C_{10f} = 1.2\,,\quad C_{11f} = 4.28\,,\quad F = 1 + 9II + 27III\,.$$

Thus, only the two independent empirical coefficients C_{10f} and C_{11f} are needed to describe the return to isotropy. An analysis of the calculation results revealed that use of the coefficient C_1 as a function of the invariant F permits a correct description of the distributions of $\Phi_{(1)xx}/\varepsilon$ and $\Phi_{(1)yy}/\varepsilon$ illustrated in Fig. 1 and introduction of the second term in relation (8) allows for differences between the components of the tensor $\Phi_{(1)ij}/\varepsilon$.

Figure 3 compares DNS data with results predicted for $\Phi_{(1)ij}$ from Eq. (8) and from models of [1-3]. Given below are the mean errors in the description of the DNS data ($\delta = \nabla\Phi_{(1)ij}/P$) using relations and empirical coefficients of [1-3]:

the model of Launder, Reece, and Rodi [1]: $\delta = 24\%$;

the model of Shih and Lumley [2]: $\delta = 55\%$;

the model of Speziale, Sarkar and Gatski [3]: $\delta = 58\%$;

model (8): $\delta = 3\%$.

It is clear from Fig. 3 that the turbulence models of [1-3] produce especially large errors in describing the wall region of the boundary layer for $Y^+ < 60$. To obtain acceptable results in this region, in [1] it was assumed

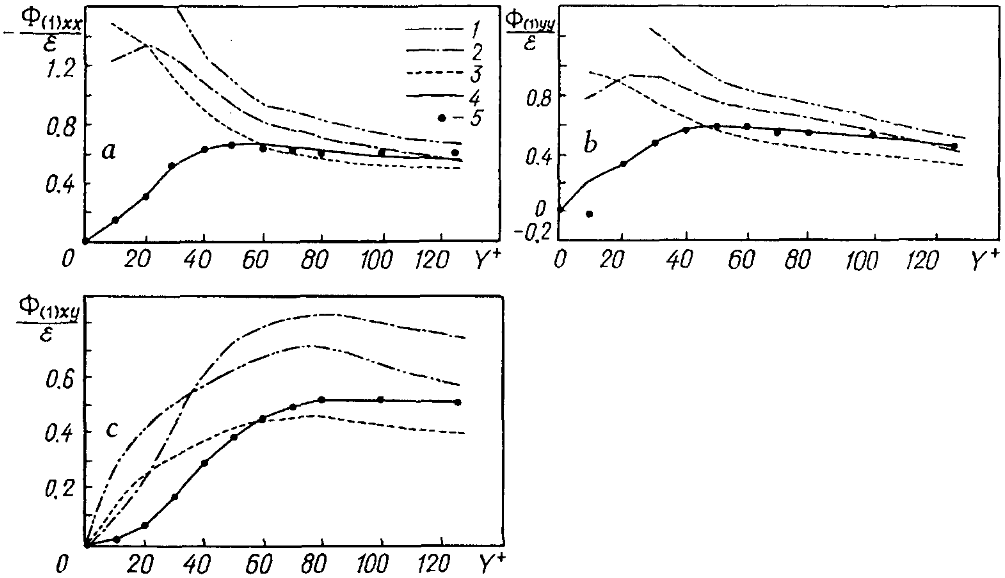

Fig. 3. Comparison of calculation results with DNS data for tyhe components $\Phi_{(1)xx}/\varepsilon$ (a), $\Phi_{(1)yy}/\varepsilon$ (b), and $\Phi_{(1)xy}/\varepsilon$ (c): 1) equation of [3]; 2) [2]; 3) [1]; 4) calculation from Eq. (8); 5) DNS data.

that the wall exerts a strong influence on the energy redistribution via pressure oscillations, and a procedure for taking this influence into account with the aid of special wall functions was devised. However, the DNS results did not confirm this assumption, and its authors rejected it in [4]. Subsequent studies [2, 3] did not employ wall functions. Study [3] suggested that the coefficient C_1 can depend on the relation of the generation of turbulent kinetic energy to the dissipation rate. However, in no way does such a connection follow from expressions (3) and (4). Furthermore, as is seen from Fig. 3, the accuracy of describing the return of turbulence to an isotropic state using equations of [3] is not satisfactory, especially in the wall region of the boundary layer.

The data presented indicate that the proposed model allows a fairly accurate calculation of the distributions of the correlation that describes the approach of turbulence to an isotropic state. It can be assumed that the high accuracy in describing all three distributions plotted in Fig. 3 adequately reflects the mechanism of the return of turbulence to isotropy not only for a special case – a developed flow in a plane-parallel duct – but also for other flows. To verify this assumption, the proposed model should be compared with results of a direct numerical simulation of rotational and some other flows. We did not manage to do this here because the published information is incomplete.

A comparison of DNS data with results predicted using model (8) demonstrates that the proposed model adequately describes the approach of turbulence to an isotropic state in the wall region of flow, where the turbulence is very close to a two-dimensional state, i.e., is highly anisotropic. Such turbulence is frequently observed in stratified and rotational flows. Previously the description of the return of highly anisotropic turbulence to an isotropic state with a sufficient degree of detail was not successful. Model (8) will presumably permit this.

Generally, in solving Eq. (1) the boundary conditions for R_{ij} have to be specified not on the wall but near it instead, for example, at $Y^+ = 30$. To define these conditions, additional conditions and constants are introduced, which complicates the problem. It is seen from Fig. 3 that model (8) is applicable throughout the boundary layer, including the immediate vicinity of the wall with $Y^+ < 30$, which corresponds to a laminar sublayer and a transitional region. If the approximations for ε_{ij} and D_{ij}, just like relation (8), will be applicable to $Y^+ \to 0$, then the specification of the boundary conditions for Eq. (1) will be simplified.

It should be noted that, before the results of a direct numerical simulation appeared, obtaining a unique form of the dependence of the coefficients C_1 and C_2 on the scalar invariants was barely practicable. This is explained by the fact that approximate expressions that contained a large number empirical coefficients were written first for all the unknown correlations Φ_{ij}, ε_{ij}, and D_{ij}. Then, parametric calculations of the flow, for which reliable experimental data as to the mean flow parameters and the Reynolds stresses were available, were performed. In the calculations the coefficients were selected such that the results were in favorable agreement with the experimental data. The errors stemming from incorrect writing of an approximate expression, for example, for the term $\Phi_{(1)ij}$, could always be compensated by the selection of the coefficients in the approximations for other correlations.

The appearance of DNS data for a duct flow, a boundary-layer flow, and stratified and rotational flows radically changes the situation, since they can be used to compare each correlation or a component of it with relevant approximate relations substantiated theoretically. Under these conditions, the probability of obtaining an unambiguous result increases by many times. To a certain extent the current study validates this assertion.

Thus, results of a direct numerical simulation are reasonable to use to develop turbulence models of the second order. These models are much simpler, require a hundredth the time, and can be applied to problems with an intricate flow geometry. Based on DNS data a relation is derived that approximates the transition of initially anisotropic turbulence to an isotropic state, although the approximation is somewhat more complex than the relations proposed previously, since it involves the second and third invariants of the tensor of Reynolds stress anisotropy. Utilization of the developed approximation improves by many times the accuracy in describing the approach of turbulence to an isotropic state in the wall region of flow and in flows with a high degree of turbulence anisotropy. The results of the study are suitable for numerical calculations based on turbulence models of the second order.

NOTATION

$R_{ij} = \langle u_i u_j \rangle$, single-point correlation of velocity oscillations; $F_{ij} = \langle u_i f_j \rangle$ and $F_{(\tau)i} = \langle \tau f_i \rangle$, generation terms due to the effect of the external force; $P_{ij} = -(R_{ik}U_{kj} + R_{jk}U_{ki})$, term of generation of Reynolds stresses by the mean-velocity gradient; $P_{(\tau)i} = -(\Gamma_k U_{i,k} + R_{ik}T_{,k})$, term of generation of heat fluxes by the mean-velocity and temperature gradients; $\Phi_{ij} = (\langle pu_{i,j} \rangle + \langle pu_{j,i} \rangle)/\rho$, term containing the correlations of strain rates with pressure oscillations; $\varepsilon_{ij} = \nu \langle u_{i,k} u_{k,j} \rangle$, dissipation term; $D_{ij} = [\langle u_i u_j u_k \rangle + (\langle pu_i \rangle \delta_{ik} + \langle pu_j \rangle \delta_{ik}) - \nu \langle u_i u_j \rangle_{,k}]_{,k}$, diffusion term; $\Phi_{(\tau)i}$, $\varepsilon_{(\tau)i}$, and $D_{(\tau)i}$, corresponding terms in the equation for heat fluxes; u_i, τ, f_i, and p, oscillations of the velocity, temperature, external force, and pressure; ρ, density; ν, kinematic viscosity; δ_{ij}, Kronecker symbol; angle brackets denote averaging; a comma in front of a subscript denotes differentiation; $K = R_{ii}/2$, turbulent kinetic energy; $\varepsilon = \varepsilon_{ii}$, rate of dissipation of K; $b_{ij}^2 = b_{ik}b_{kj}$; $b_{ij}^3 = b_{ik}b_{km}b_{mj}$; II and III, scalar invariants: $II = -b_{ik}b_{ki}/2$, $III = b_{ik}b_{km}b_{mi}/3$; F, scalar invariant determining the degree of turbulence anisotropy; $\Phi_{(1)xx}$, $\Phi_{(1)yy}$, and $\Phi_{(1)xy}$, components of the tensor $\Phi_{(1)ij}$; $Y^+ = yu_0/\nu$, dimensionless distance from the wall; u_0, dynamic velocity on the wall.

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